Application of short multiplication formulas for squares – proofs

1. Prove that the number $13^8 - 7^8$ is divisible by 120.

2. Prove that the number $19^{16} - 9^{16}$ is divisible by 280.

3. Prove that the number $11^{32} - 7^{32}$ is divisible by 72.

4. Prove that the inequality
$$\frac{a+b}{2} \leqslant \sqrt{\frac{a^2+b^2}{2}}$$
is true for every real numbers a and b.

5. Prove that the inequality
$$\frac{2}{\frac{1}{a}+\frac{1}{b}} \leqslant \frac{a+b}{2}$$
is true for every positive numbers a and b.

6. Prove that the inequality
$$\frac{2}{\frac{1}{a}+\frac{1}{b}} \leqslant \sqrt{ab}$$
is true for every positive numbers a and b.

7. Prove that the inequality
$$2x^2 - 2xy - 2x + y^2 + 1 \geqslant 0$$
is true for every real numbers x and y.

8. Prove that the inequality
$$y^2 + 2x^2 - 2xy - 6x + 9 \geqslant 0$$
is true for every real numbers x and y.

9. Prove that the inequality
$$5x^2 + y^2 + 4 \geqslant 4xy + 4x$$
is true for every real numbers x and y.