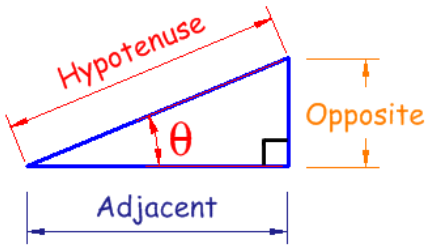
1.7 GEOMETRY ON THE PLANE (2) - TRIANGLES

THEORY

1. Basic Vocabulary

A triangle is one of the basic shapes of geometry. It is a 3-sided **polygon**. Every triangle has 3 sides and 3 angles. The 3 angles always add to 180°

- Equilateral, Isosceles and Scalene: We distinguish three special names given to triangles that tell how many sides (or angles) are equal.
  - **Equilateral Triangle**: 3 equal sides and 3 equal angles (always 60°)
  - **Isosceles Triangle**: 2 equal sides and 2 equal angles
  - **Scalene Triangle:** no equal sides and no equal angles
- Acute, Right and Obtuse: Triangles can also have names that tell you what type of angle is inside.
  - **Acute Triangle:** all angles are less than 90°
  - **Right Triangle:** has a right angle (90°)



  - **Obtuse Triangle:** has an angle more than 90°
- **Vertex** – The vertex (plural: vertices) is a corner of the triangle. Every triangle has three vertices.
- **Base** – The base of a triangle can be any one of the three sides, usually the one drawn at the bottom. You can pick any side you like to be the base. Commonly used as a reference side for calculating the area of the triangle. In an isosceles triangle, the base is usually taken to be the unequal side.
- **Altitude** – The altitude of a triangle is the perpendicular from the base to the opposite vertex (The base may need to be extended). Since there are three possible bases, there are also three possible altitudes. The three altitudes intersect at a single point, called **the orthocenter of the triangle**.
- **Perpendicular bisector line** – is a line perpendicular to the side and passing through its midpoint.
- **Bisector of an angle** – is a line or **ray** that divides an angle into two congruent angles.
- **Median** – The median of a triangle is a line from a vertex to the midpoint of the opposite side. The three medians intersect at a single point, called **the centroid of the triangle**.
- **Area** – Usually called "half of base times height", the area of a triangle is given by the formula below.

$$A = \frac{ah}{2}$$

where a is the length of the base , h is the length of the corresponding altitude.

- **Perimeter** – The distance around the triangle. The sum of its sides.
- **Interior angles** – The three angles on the inside of the triangle at each vertex.
- **Exterior angles** – The angle between a side of a triangle and the extension of an adjacent side.

2. Properties

Theorem 1 (The triangle inequality theorem) *Any side of a triangle is always shorter than the sum of the other two sides.*

Theorem 2 (Relationship of sides to interior angles in a triangle) *In any triangle:*
1. The shortest side is always opposite the smallest interior angle
2. The longest side is always opposite the largest interior angle

Theorem 3 (Pythagorean) *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

Definition 1 *Two triangles are said to be* ***congruent*** *if their sides have the same length and angles have same measure.*

- If all the three sides of one triangle are equivalent to the corresponding three sides of the second triangle, then the two triangles are said to be congruent by **SSS rule**.
- If any two sides and angle included between the sides of one triangle are equivalent to the corresponding two sides and the angle between the sides of the second triangle, then the two triangles are said to be congruent by **SAS rule**.
- If any two angles and side included between the angles of one triangle are equivalent to the corresponding two angles and side included between the angles of the second triangle, then the two triangles are said to be congruent by **ASA rule**.

Definition 2 *Two triangles are said to be* ***similar*** *if they have equal angles and* ***proportional sides****.*

Theorem 4 (Basic Proportionality Theorem form Thales) *(can be abbreviated as BPT) If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in* ***proportion****.*