Geometric interpretation of simultanous equations

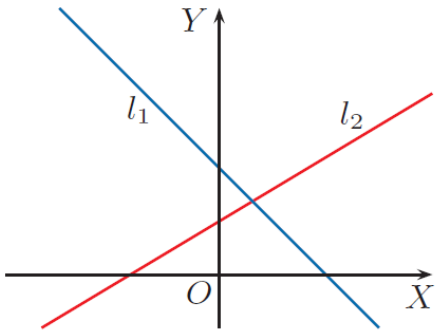
Let us consider the lines l_1 and l_2 described by the equations of the system:

The coordinates of the points (x, y) that belong simultaneously to both lines are solutions of

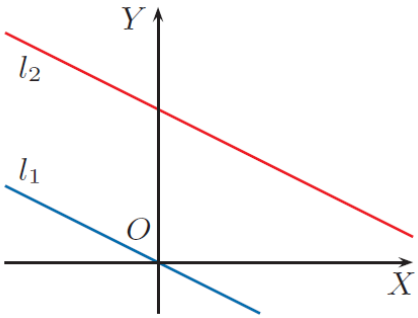
$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

that system. One of the following situations can occur.

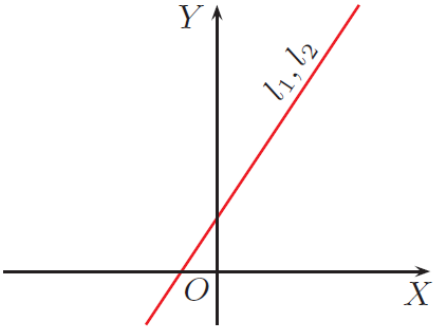
A definite system of equations (has one solution) – the lines intersect at a single point.



A contradictory system of equations (has no solutions) – the lines are parallel and distinct.



An indefinite system of equations (has infinitely many solutions) – the lines coincide.

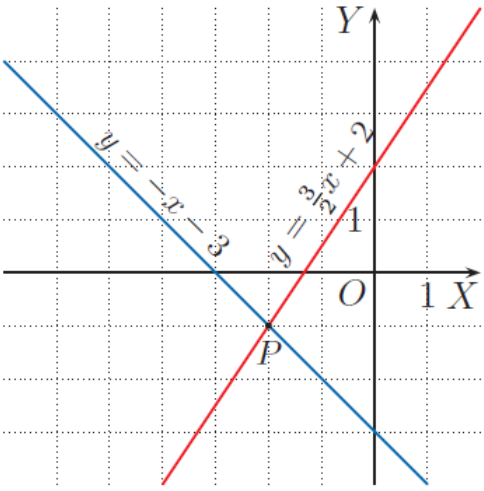


Example 1

Solve the system of equations graphically.

$$\begin{cases} x + y = -3 \\ 3x - 2y = -4 \end{cases}$$

The first equation, after transforming to slope-intercept form, is $y = -x - 3$, and the second $y = \frac{3}{2}x + 2$. Sketch both lines and read the coordinates of their intersection: P(−2,−1).



The solution to the system of equations is therefore the pair of numbers: $x = -2$, $y = -1$ (the correctness of the solution can be checked by substituting them into the system of equations).

Example 2

Solve the system of equations graphically.

$$\begin{cases} x - 3y = -3 \\ 2x - 6y = -6 \end{cases}$$

Both equations describe the same line with slope $y = \frac{1}{3}x + 1$.

Therefore, the system *has infinitely* many solutions. These are all pairs of numbers: $(x, \frac{1}{3}x + 1)$, where $x \in R$.

Solutions to this system are, for example, the pairs (−3, 0), (0, 1), and (3, 2).

Give three other pairs of numbers that are solutions to this system of equations.

Exercise 1

Solve the system of equations graphically and algebraically.

a) $\begin{cases} 2x - y = -4 \\ x + y = 1 \end{cases}$ b) $\begin{cases} 4x - y = 3 \\ -x + 2y = 8 \end{cases}$ c) $\begin{cases} x - y - 3 = 0 \\ 3x + y = 1 \end{cases}$

Exercise 2

Solve the system of equations graphically.

a) $\begin{cases} x - y = -1 \\ -x + y = 3 \end{cases}$ b) $\begin{cases} 2x - y = 6 \\ 0{,}5y - x = -3 \end{cases}$ c) $\begin{cases} -3x + 2y = 2 \\ 6x - 4y = 8 \end{cases}$

Exercise 3.

Determine whether the system of equations is determinate, indeterminate, or inconsistent.

a) $\begin{cases} 2x - y = 1 \\ 4x - 2y = 6 \end{cases}$ b) $\begin{cases} 3x - 2y = -1 \\ -3x + 2y = 1 \end{cases}$ c) $\begin{cases} 2x + y = 1 \\ -3x + y = -4 \end{cases}$

HOMEWORK

Solve the system of equations graphically.

a) $\begin{cases} y = \frac{1}{3}x + 3 \\ y + x = 7 \end{cases}$ c) $\begin{cases} 3x - 2y = 4 \\ 3x - y = 5 \end{cases}$ e) $\begin{cases} 2x - 3y = 6 \\ -\frac{4}{3}x + 2y = -4 \end{cases}$

b) $\begin{cases} y = -2x + 2 \\ 4y + 3x = -12 \end{cases}$ d) $\begin{cases} y - 2x = 4 \\ 9x - 2y = -3 \end{cases}$ f) $\begin{cases} 3y + x = 9 \\ y + 3 = -\frac{1}{3}x \end{cases}$