1.3 The set of integer numbers

1. `Integers`

   - $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ – the set of integer numbers
   - $\mathbb{Z}_+ = \{1, 2, 3, 4, \ldots\}$ – the set of **positive integers**
   - $\mathbb{Z}_- = \{\ldots, -4, -3, -2, -1\}$ – the set of **negative integers**

   Therefore
   $$\mathbb{Z} = \mathbb{Z}_- \cup \{0\} \cup \mathbb{Z}_+$$

2. `Divisibility`

   - An integer number n is divisible by an integer number $m \neq 0$ if and only if, there exists such a number $k \in \mathbb{Z}$, that
     $$n = k \cdot m.$$
     The fact that m is the devisor of n is written as $m|n$.
   - Every integer number n can be written as
     $$n = k \cdot m + r$$
     where $0 \leqslant |r| < m$ and r is a **remainder** of the division n by m.
   - $2k$ – a number that is divisible by 2 – **an even number**, $k \in \mathbb{Z}$
   - $2k \pm 1$ – a number that is not divisible by 2 – **an odd number**, $k \in \mathbb{Z}$
   - $n = 3k + 2$ – a number n divided by 3 gives a remainder of 2.
   - $n = 4k + 1$ – a number n divided by 4 gives a remainder of 1.
   - (etc)

EXERCISES

1. Write as an algebraic expression
   (a) The sum of three consecitive odd integer numbers.
   (b) The sum of four consectutive even integer numbers.
   (c) The difference between the number divisible by three and the number divisible by four.
   (d) The sum of the number that divided by 4 gives the remainder of 3 and the number that divided by 5 gives the remainder of 2.
2. Write symbolically using k, where $k \in \mathbb{Z}$.
   (a) three consecutive odd numbers and the lowest of which is equal to $2k - 3$,
   (b) three consecutive even numbers and the greatest of which is equal to $2k + 6$,
   (c) three numbers when divided by 5 give a remainder of 4 and the lowest of which is equal to $5k - 1$,
   (d) the product of three consecutive integer numbers,
   (e) the opposite to the number $3k + 2$.
3. The product of two consecutive even integer numbers is equal to the square of the smaller number. Find the numbers.
4. The product of two consecutive odd integer numbers is 34 smaller than the square of the lower smaller number. Find the numbers.
5. Prove that:
   (a) the sum of four consecutive even integers is divisble by 4,
   (b) the product of two consecutive integers is divisible by 2,
   (c) the sum of three consecutive integers is divisible by 3,
   (d) the sum of four consecutive odd numbers is divisible by 8.