LINEAR FUNCTION
THEORY & EXERCISES

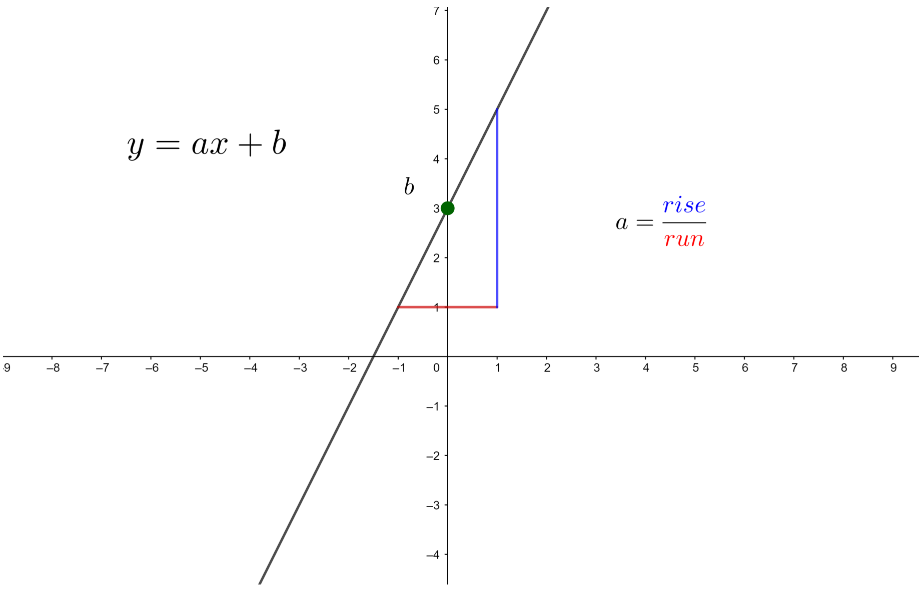
1. Linear function

Definition 1 (SLOPE - INTERCEPT FORM) *A linear function* $f : \mathbb{R} \to \mathbb{R}$ *is a function of the form* $f(x) = ax + b$*, where* a *and* b *are real numbers.*

a – slope (or gradient)
b – y-intercept

The slope of a line refers to the slant or inclination of the line. The slope is **the ratio of the vertical change to the horizontal change between two points on the line**. The slope can also be called the **rise over run ratio** because it tells you how many spaces to move up or down and how many spaces to move to the right. A positive sign will move the line up and a negative sign will move the line down. One important thing to remember is that the run will always be to the right, regardless of the sign.

$$a = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x}$$



2. Properties of the linear function

(a) `Increasing, Decreasing, Constant.`

- $a > 0$ - the function f is increasing on $\mathbb{R}$
- $a < 0$ - the function f is decreasing on $\mathbb{R}$
- $a = 0$ - the function f is constant on $\mathbb{R}$

(b) `Zeros of the linear function`

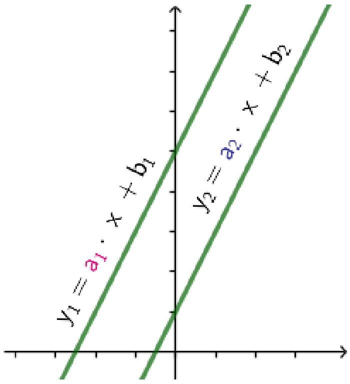
- if $a \neq 0$, then the linear function has only one zero $x_0 = -\frac{b}{a}$
- if $a = 0$ and $b \neq 0$ the the linear function has no zeros.

- if $a = b = 0$ then the linear function has infinite number of zeros.

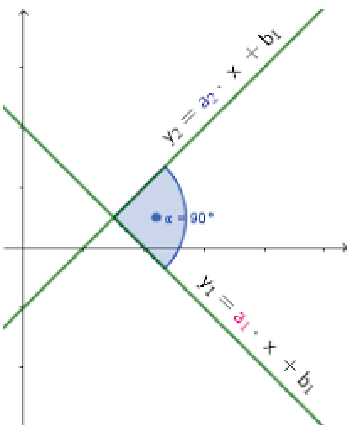
Condition	Form	No. of zeros
$a \neq 0$	$y = ax + b$	one - $x_0 = -\frac{b}{a}$
$a = 0,\ b \neq 0$	$y = b$	none
$a = b = 0$	$y = 0$	∞

(c) `Parallel & Perpendicular lines`

Theorem 1 *The graphs of functions $f(x) = a_1x + b_1$ and $g(x) = a_2x + b_2$ are* ***parallel*** *if $a_1 = a_2$*



Theorem 2 *The graphs of functions $f(x) = a_1x + b_1$ and $g(x) = a_2x + b_2$ are* ***perpendicular*** *if $a_1 \cdot a_2 = -1$ and $a_2 = -\frac{1}{a_1}$ (a_2 is the* ***negative reciprocal*** *of a_1)*



3. Practical Hints

(a) To find the equation of a line when only points or a slope is given, use **the point-slope form** of a linear equation formula:

$$y = a(x - x_1) + y_1$$

where a is the slope of the line and (x_1, y_1) is the point on the line.

(b) The formula to find the slope of a line passing through the points (x_1, y_1) and (x_2, y_2) is:

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

(c) To find the equation of the line when two points (x_1, y_1) and (x_2, y_2) are given, use the formula:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

4. EXERCISES

(a) Use the point-slope formula to find an equation for the line through the point $(2, 1)$ with slope $\frac{1}{3}$.

(b) Use the point-slope formula to find an equation for the line through the points $(1, 2)$ and $(5, 8)$.

(c) Let f be a linear function, and suppose that $f(1) = 5$ and f has slope $-1/2$. Find a formula for $f(x)$.

(d) Let f be a linear function, and suppose that $f(-1) = 4$ and $f(4) = 8$. Find a formula for $f(x)$.

(e) Let f be a linear function, and suppose that $f(-3) = -1$ and $f(3) = 5.5$. Find $f(5)$.

(f) Write the equation in slope-intercept form of the line that is parallel to the graph of each equation and passes through the given point P.

i. $y = 3x + 6$; $P(4, 7)$
ii. $y = x - 4$; $P(-2, 3)$
iii. $y = \frac{1}{2}x + 6$; $P(4, -5)$
iv. $y = -2x + 4$; $P(-1, 2)$

(g) Write the equation in slope-intercept form of the line that is perpendicular to the graph of each equation and passes through the given point P.

i. $y = -5x + 1$; $P(2, -1)$
ii. $y = 2x - 3$; $P(-5, 3)$
iii. $y = \frac{1}{2}x - 1$; $P(4, -1)$
iv. $y = -2x + 3$; $P(-1, -1)$

(h) A meteorologist is using a weather balloon to measure the air temperature at high altitudes. At the time of the measurement, the air temperature at sea level was approximately $21°C$, and the air temperature at an altitude of 4.0 km was approximately $-5°C$.
(a) How quickly is the air temperature decreasing with altitude?
(b) Find an approximate linear formula for the air temperature at an altitude of h kilometers.