1.6 Powers with the integer exponent

1. `Powers with a natural exponent`
   A power with a natural exponent indicates repeated multiplication of a base number by itself. The natural exponent (or power) represents how many times the base is used as a factor in the multiplication. For example, in the expression 4^3, **the base** is 4, and **the exponent** is 3. This means 4 is multiplied by itself three times: $4 \cdot 4 \cdot 4 = 64$. In general:

$$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}, \quad \text{where} \quad a \in \mathbb{R}, \quad n \in \mathbb{N}.$$

2. `Powers with a negative exponent`
   A negative exponent indicates **the reciprocal** of the base raised to the corresponding positive exponent. In simpler terms, if you have a number raised to a negative power, you can rewrite it as 1 divided by that number raised to the positive version of the exponent.

$$a^{-n} = \left(\frac{1}{a}\right)^n, \quad \text{where} \quad a \in \mathbb{R}\backslash\{0\}, \quad n \in \mathbb{N}.$$

3. `Exponent Rules`

$$a^n \cdot a^m = a^{n+m} \tag{1}$$

$$a^n : a^m = a^{n-m} \tag{2}$$

$$(a^n)^m = a^{n \cdot m} \tag{3}$$

$$a^n \cdot b^n = (a \cdot b)^n \tag{4}$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \tag{5}$$

EXERCISES

1. Simplify

(a) $\dfrac{(2x^3y^{-2})^2}{4x^2y}$

(b) $\left(\dfrac{a^{-1}b^2}{c^3}\right)^3 \cdot \left(\dfrac{c}{b}\right)^2$

(c) $\dfrac{x^{-2}y^3}{(x^3y^{-1})^2}$

(d) $\left(3a^{-2}b\right)^2 \cdot a^3b^{-4}$

(e) $\left(\dfrac{2^3 \cdot 5^{-2}}{10}\right)^2$

(f) $\dfrac{(x^2y^{-3})^3 \cdot x^{-4}y}{x^3y^2}$

(g) $\left(\dfrac{m^{-2}n^3}{p^0}\right)^2 \cdot m^4n^{-1}$

(h) $\dfrac{4a^2b^{-3}}{(2a^{-1}b)^2}$

(i) $\dfrac{(x^0y^{-1})^2}{x^{-3}y^3}$

(j) $(2x^{-2}y)^3 \cdot \left(\dfrac{x^4}{y^2}\right)$