Quadratic function

1. Write the factored form and the vertex form of the function $f(x) = x^2 + 6x + 10$. What is the range and axis of symmetry of the graph $y = f(x)$.

   Ans. $f(x) = (x+3)^2 + 1$, no factored form, $f(D) = [1; +\infty), x = -3$

2. For the function $f(x) = 2x^2 + 4x + 5$ find:

   a) the coordinates of the vertex (turning point) of the parabola.
   b) the vertex form of the function f.
   c) the range of the function f.
   d) the lowest value and intervals on which the function increases and decreases.

   Ans. a) $V(-1,,3)$, b) $f(x) = 2(x+1)^2 + 3$ c) $f(D) = [3; +\infty)$ d) $y_{min} = 3$) $V(-1,,3)$, $f \uparrow: [-1; +\infty)$ $f \downarrow: (-\infty; -1]$

3. The point $V(4,0)$ is the turning point of the graph of the function $f(x) = 2x^2 + bx + c$. Find b, c.

   Ans. $f(x) = 2(x-4)^2 = 2x^2 - 16x + 32$

4. The greatest value of the function f is equal to 4 for $x = -3$. The point $P(-1,3)$ lies on the graph of the function f. Find the formula of the function f.

   Ans. $f(x) = -0.25(x+3)^2 + 4$

5. The quadratic function is given by the formula $f(x) = ax^2 + bx + c$. Find a, b, c, knowing that the positive values of f are in the interval $(0,12)$ and the greatest value of the function f is equal to 9.

   Ans. $a = -\frac{1}{4}$, $b = 3, c = 0$

6. Find the value of coefficient b of the function $f(x) = x^2 + bx + c$, if $f(-1) = f(3) = 1$.

   Ans. $b = -2$,

7. Let g be a quadratic function. The line $x = 3$ is the axis of symmetry of the parabola $y = g(x)$ and $x_1 = 1$ is one of the zeros of the function g. Point $P(2, -6)$ lies on that parabola. Write the factored form and the standard form of the function g.

   Ans. $g(x) = 2(x-1)(x-5)$

8. Write the general form of the quadratic function f, if point $V(-3,4)$ is the vertex of the parabola $y = f(x)$ and point $A(-6, -5)$ lies on the graph of function f.

   Ans. $f(x) = -x^2 - 6x - 5$

9. The graph of function f passes through points $(1,1)$ and $(-1, -1)$. Prove that the function has two zeros, product of which is equal to -1.

10. Given the function $f(x) = x^2 - 6x$, find the smallest integer value of c, such that the function $k(x) = f(x) + c$ cannot be written in a factored form.

Ans. $c = 10$

11. Find the general form of the quadratic function, if the graph of the function passes through points $A(-2, -3)$, $B(0, -3)$, $C(2,5)$

Ans. $f(x) = x^2 + 2x - 3$