Theoretical probability- basic level

1. **Random experiment**: is a process or activity which produces a number of possible outcomes. The outcomes which result cannot be predicted with absolute certainty.

2. **Sample space** Ω: is a list of all possible outcomes of the experiment. The outcomes must be mutually exclusive and exhaustive. Mutually exclusive means they are distinct and non-overlapping. Exhaustive means complete.

3. **Event** A: is a subset of the sample space. An event can be classified as a SIMPLE EVENT or COMPOUND EVENT.

   (a) Ω is referred to as THE SURE EVENT.

   (b) $\emptyset$ is referred to as the IMPOSSIBLE EVENT.

   (c) UNION OF EVENTS: For any two events A and B, the event $A \cup B$ consists of all outcomes that are either in A or in B, meaning that $A \cup B$ is realized if either A or B occurs.

   (d) INTERSECTION: For any two events A and B, the event $A \cap B$ consists of all outcomes that are both in A and in B, meaning that $A \cap B$ is realized if both A and B occur.

   (e) A and B are said to be MUTUALLY EXCLUSIVE if $A \cap B = \emptyset$: $A \cap B$ is the impossible event, meaning that A and B cannot both occur in the same time.

   (f) For any event $A \subset \Omega$, we define the event A' or (A^c), referred to as the COMPLEMENT OF A, to consist of all outcomes in the sample space Ω that are not in A, meaning that A' is realized if A does not occur. Note that $A \cap A' = \emptyset$ and $A \cup A' = \Omega$.

4. **The theoretical probability (also known as Classical approach) of an event** $A \subset \Omega$ is defined as the number of ways the event A can occur divided by the number of events of the sample space Ω. Using mathematical notation, we have

$$P(A) = \frac{|A|}{|\Omega|}$$

   where $|A|$ is the number of ways the event can occur and $|\Omega|$ represents the total number of events in the sample space.

1. A random experiment consists of rolling a symmetrical six-sided dice twice, We write down outcomes in a sequence obtaining a two-digit number.
   Calculate the probability of event A that the two-digit number obtained will be odd and divisible by 3. Write down your calculations.

   Ans: $\frac{1}{6}$

2. There are two sets: $X = \{-3, -2, -1, 0, 1, 2\}$ and $Y = \{-2, -1, 0, 1\}$. We draw one number from the set X and one number from the set Y forming an ordered pair of numbers (x, y), where x is the number from the set X and y is the number from the set Y. Calculate the probability of event $A-$ a drawn pair of numbers (x, y) satisfy the condition $x \cdot y \geqslant 0$. Write down your calculations.

   Ans: $\frac{17}{24}$

3. There is a pair of balls in each of three boxes. One ball is red and one ball is blue. We draw one ball from each box. Let p mean the probability of the event that exactly two of the three drawn balls are red. Then:
   A. $p = \frac{1}{4}$ B. $p = \frac{3}{8}$ C. $p = \frac{1}{2}$ D. $p = \frac{2}{3}$

   Ans: B

4. 115 people were asked what type of ticket they purchased at the ticket office. The results of the survey are presented in the table below.

Type of tickets purchased	Number of people
discounted	76
standard	41

   Note: 27 of the respondents purchased both types of tickets.
   Calculate the probability of the event that a randomly selected respondent did not buy any ticket. Give the result as an irreducible fraction.

   Ans: $\frac{5}{23}$

5. A symmetrical cubic dice is cast twice. Calculate the probability of the event A that the side with five dots appears at least once.

   Ans: $\frac{11}{36}$

6. The numbers $1, 2, 3, 4, 5, 6, 7, 8$ are randomly arranged in a row. Calculate the probability that the sum of any two consecutive numbers is odd. Give your answer as an irreducible fraction.

Ans: $\frac{1}{35}$

7. A symmetrical cubic dice is cast four times. Calculate the probability of the event that the product of the numbers obtained in all four rolls is equal to 60.

Ans: $\frac{1}{18}$

8. Three students (W, X, Y) draw lots for seats in a four-seat row on a plane. Determine the sample space and events: A – the student W sits next to the student X, B – the student X sits on the left of the student Y. Calculate $P(A)$, $P(B)$.

Ans: $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$

9. We draw two numbers from the set $\{1, 2, 3, ..., 7\}$ such that the same numbers can apper twice. (drawing with repetitions) Calculate the probability of the event that the sum of selected numbers is divisible by 3.

Ans: $\frac{16}{49}$

10. The test shows that 30% of light bulbs made by a certain factory are defective. A customer bought one randomly selected light bulb (without checking it). A few moments later, he decided to buy another one. Is the probability that the customer will receive two good light bulbs greater than 0.5? Justify your answer by making appropriate calculations.

Ans: No

11. We draw one number from the set of all two-digit natural numbers,. Calculate the probability of the event $A-$ the drawn number has a tens digit from the set $\{1, 3, 5, 7, 9\}$, and at the same time a units digit from the set $\{0, 2, 4, 6, 8\}$.

Ans: $\frac{5}{18}$

12. There are 10 people and 50 big rooms. Suppose that each person has equal probability $\frac{1}{50}$ to go to any one of the 50 rooms. Let an event A - for 10 specified rooms, each of these 10 rooms has exactly one person. Find out the probability of event A.

Ans: $\frac{10!}{50^{10}}$

13. A jar contains four coins: a nickel, a dime, a quarter, and a half-dollar. Three coins are randomly selected from the jar.

    (a) List all the simple events in the sample space.

    (b) This is a typical example of the classical probability. What is the probability that the selection will contain the half-dollar?

    (c) What is the probability that the total amount drawn will equal 0.6 dollar or more?

Ans: (a) $\Omega = \{NDQ, NDH, DQH, NQH\}$ (b) $\frac{3}{4}$, (c) $\frac{3}{4}$

14. A random experiment consists in a simultaneous toss of two coins and a cubic dice. The result of tossing a coin may be heads or tails. Each of the six faces of the dice contains a different number of dots. The number of the dots belongs to the set $\{1, 2, 3, 4, 5, 6\}$. Calculate the probability, of events:

    (a) The result of the experiment is two tails and a face with six dots.

    (b) The result of the experiment is two heads and a face with an even number.

Ans: (a) $\frac{1}{24}$, (b) $\frac{1}{8}$