

1.4 Rational numbers

1. The set of rational numbers

$\mathbb{Q} = \left\{ \frac{a}{b}, a \in \mathbb{Z}, b \in \mathbb{Z} \setminus \{0\} \right\}$ – the set of rational numbers

A **rational number** is a number that can be written as:

$$\frac{a}{b} \quad (1)$$

where $a, b \in \mathbb{Z}$ and $b \neq 0$; a – **a numerator**, b – **a denominator**. The bar between the numbers is called **the fraction bar**. Such numbers are called **fractions**. A fraction whose numerator is less than the denominator is called **a proper fraction**, otherwise it is called **an improper fraction**. A fraction in which there is no common factor, except 1, in its numerator and denominator is called a fraction in **the simplest** or **lowest** form. Fractions with same denominators are called **like fractions** and if the denominators are different, then they are called **unlike fractions**.

2. Examples

- $\frac{1}{3}$ – the proper fraction
- $2\frac{5}{7} = \frac{19}{7}$ – the improper fraction
- 0 because $0 = \frac{0}{b}$, where $b \neq 0$
- -4 because $-4 = \frac{-40}{10}$

3. Decimals

Every fraction can be expressed as **a decimal fractions**. Rational numbers have two types of such expansion **terminating** or **non-terminating repeating** expansion.

(a) Terminating Decimals:

These decimals have a finite number of digits after **the decimal point**, meaning they end. For example: 0.25, 0.75, 1.5, 2.15. They can always be expressed as a fraction where the denominator is a power of 10, e.g. $0.25 = \frac{25}{100} = \frac{1}{4}$

(b) Non-Terminating Repeating (or Recurring) Decimals:

These decimals have an infinite number of digits after the decimal point, and a block of digits repeats endlessly. For example: $0.3333\ldots = 0.(3)$ (where the 3 repeats), $0.142857142857\ldots = 0.(142857)$ (where the block 142857 repeats).

4. Conversion decimals to a fractions

(a) for terminating decimals

Example:

$$2.43 = 2\frac{43}{100} = \frac{243}{100}$$

(b) for non-terminating recurring decimals

Example: Lets take $0.(18) = 0.181818\ldots$. Assume that $0.(18) = x$. Multiply it by 100, then $18.(18) = 100x$. Thus we have

$$100x - x = 18.(18) - 0.(18)$$

$$99x = 18 \quad | : 99$$

and we have $x = \frac{18}{99} = \frac{2}{11}$.

EXERCISES

1. Find a rational number that is between numbers $\frac{3}{7}$ and $\frac{4}{7}$.
2. Find three rational numbers that are between numbers $\frac{7}{9}$ and $\frac{8}{9}$.
3. Find five rational numbers that are between numbers $\frac{1}{3}$ and $\frac{2}{3}$.
4. Find three rational numbers that are between numbers 0.28 and $\frac{9}{25}$.
5. Find three rational numbers that are between numbers 0.25 and $\frac{2}{7}$.
6. Convert the following decimals into fractions:
 - (a) 0.(16)
 - (b) 0.(123)
 - (c) 0.3(4)
 - (d) 0.23(5)
 - (e) 1.2(15)
 - (f) -3.32(4)