# SYSTEMS OF EQUATIONS

## MAIN METHODS OF SOLVING

Simultaneous equations

A SYSTEM OF EQUATIONS

$$\begin{cases} a_1x + b_1y = c_1 & a_1^2 + b_1^2 \neq 0, \\ a_2x + b_2y = c_2 & a_2^2 + b_2^2 \neq 0 \end{cases}$$

**A system of linear equations is a set of two (or more) linear equations that share some of the same variables.**

METHODS OF SOLVING

(a) **by substitution**

1. Solve ONE of the equations for $x$ or for $y$
(This step can be skipped if one of the equations is already solved for $x$ or $y$)

2. Substitute the result into the other equation for that variable

3. The new equation will now be all $x$ terms, or all $y$ terms. Solve the equation for the variable.

4. Plug the answer back into the original equation (the one you initially solve for $x$ or $y$) to find the other answer.

(b) **addition / elimination method**

1. Get the $x$'s and $y$'s on the same side of the equation and lined up with each other.

2. Find a common multiple for EITHER the $x$'s or the $y$'s, whichever you choose to eliminate.

3. Make one of those multiples a negative, so that the equations can be added together.

4. Once the equations are added together, one of the variables will be eliminated. Finish solving for the variable that is still present.

5. Take your answer from point 4, and plug it into either of the equations to find the other value.

# Solve the following system of equations by elimination

1.

$$\begin{cases} 2x + y = 7 \\ 2x - y = 3 \end{cases}$$

2.

$$\begin{cases} 3x - 2y = 4 \\ 3x + 2y = 16 \end{cases}$$

3.

$$\begin{cases} x + 4y = 10 \\ -x + 4y = 2 \end{cases}$$

4.

$$\begin{cases} 4x + 3y = 27 \\ 2x - 3y = -3 \end{cases}$$

5.

$$\begin{cases} 5x - y = 19 \\ 3x + y = 11 \end{cases}$$

6.

$$\begin{cases} 7x + 2y = 30 \\ 5x - 2y = 10 \end{cases}$$

7.

$$\begin{cases} 6x - 5y = 13 \\ 3x + 5y = 32 \end{cases}$$

8.

$$\begin{cases} 4x - 7y = 1 \\ 6x + 7y = 43 \end{cases}$$

9.

$$\begin{cases} 9x + 4y = 11 \\ 6x - 8y = 12 \end{cases}$$

10.

$$\begin{cases} 8x + 3y = 5 \\ 5x - 9y = 52 \end{cases}$$

Solve the following system of equations by elimination method

1.

$$\begin{cases} 2x + y = 7 \\ 2x - y = 3 \end{cases}$$

Solve the following system of equations by elimination method

9.

$$\begin{cases} 9x + 4y = 11 \\ 6x - 8y = 12 \end{cases}$$

# Solutions

**1.**

$$\begin{cases} 2x + y = 7 \\ 2x - y = 3 \end{cases}$$

Add the equations: $4x = 10 \Rightarrow x = \dfrac{10}{4} = \dfrac{5}{2}$.

$y = 7 - 2x = 7 - 2 \cdot \dfrac{5}{2} = 7 - 5 = 2.$

**Solution:** $x = \frac{5}{2},\ y = 2.$

**2.**

$$\begin{cases} 3x - 2y = 4 \\ 3x + 2y = 16 \end{cases}$$

Add: $6x = 20 \Rightarrow x = \dfrac{20}{6} = \dfrac{10}{3}$.

$3x + 2y = 16 \Rightarrow 3 \cdot \dfrac{10}{3} + 2y = 16 \Rightarrow 10 + 2y = 16 \Rightarrow y = 3.$

**Solution:** $x = \frac{10}{3},\ y = 3.$

**3.**

$$\begin{cases} x + 4y = 10 \\ -x + 4y = 2 \end{cases}$$

Add: $8y = 12 \Rightarrow y = \dfrac{12}{8} = \dfrac{3}{2}$.

$x = 10 - 4y = 10 - 4 \cdot \dfrac{3}{2} = 10 - 6 = 4.$

**Solution:** $x = 4,\ y = \frac{3}{2}.$

**4.**

$$\begin{cases} 4x + 3y = 27 \\ 2x - 3y = -3 \end{cases}$$

Add: $6x = 24 \Rightarrow x = 4.$

$4 \cdot 4 + 3y = 27 \Rightarrow 16 + 3y = 27 \Rightarrow 3y = 11 \Rightarrow y = \dfrac{11}{3}.$

**Solution:** $x = 4,\ y = \frac{11}{3}.$

**5.**

$$\begin{cases} 5x - y = 19 \\ 3x + y = 11 \end{cases}$$

Add: $8x = 30 \Rightarrow x = \dfrac{30}{8} = \dfrac{15}{4}$.

$y = 11 - 3x = 11 - 3 \cdot \dfrac{15}{4} = 11 - \dfrac{45}{4} = \dfrac{44 - 45}{4} = -\dfrac{1}{4}.$

**Solution:** $x = \frac{15}{4},\ y = -\frac{1}{4}.$

**6.**

$$\begin{cases} 7x + 2y = 30 \\ 5x - 2y = 10 \end{cases}$$

Add: $12x = 40 \Rightarrow x = \dfrac{40}{12} = \dfrac{10}{3}$.

$7 \cdot \dfrac{10}{3} + 2y = 30 \Rightarrow \dfrac{70}{3} + 2y = 30 \Rightarrow 2y = 30 - \dfrac{70}{3} = \dfrac{20}{3} \Rightarrow y = \dfrac{10}{3}.$

**Solution:** $x = \frac{10}{3},\ y = \frac{10}{3}.$

# Solutions

**8.**

$$\begin{cases} 4x - 7y = 1 \\ 6x + 7y = 43 \end{cases}$$

Add: $10x = 44 \Rightarrow x = \dfrac{44}{10} = \dfrac{22}{5}$.

$4 \cdot \dfrac{22}{5} - 7y = 1 \Rightarrow \dfrac{88}{5} - 7y = 1 \Rightarrow -7y = 1 - \dfrac{88}{5} = -\dfrac{83}{5} \Rightarrow y = \dfrac{83}{35}$.

**Solution:** $x = \frac{22}{5},\ y = \frac{83}{35}$.

**9.**

$$\begin{cases} 9x + 4y = 11 \\ 6x - 8y = 12 \end{cases}$$

Multiply the first equation by $2$: $18x + 8y = 22$.

Add to the second: $24x = 34 \Rightarrow x = \dfrac{34}{24} = \dfrac{17}{12}$.

$9 \cdot \dfrac{17}{12} + 4y = 11 \Rightarrow \dfrac{153}{12} + 4y = 11 \Rightarrow 4y = 11 - \dfrac{153}{12} = -\dfrac{7}{4} \Rightarrow y = -\dfrac{7}{16}$.

**Solution:** $x = \frac{17}{12},\ y = -\frac{7}{16}$.

**10.**

$$\begin{cases} 8x + 3y = 5 \\ 5x - 9y = 52 \end{cases}$$

Multiply the first equation by $3$: $24x + 9y = 15$.

Add to the second: $29x = 67 \Rightarrow x = \dfrac{67}{29}$.

$8 \cdot \dfrac{67}{29} + 3y = 5 \Rightarrow \dfrac{536}{29} + 3y = 5 \Rightarrow 3y = -\dfrac{391}{29} \Rightarrow y = -\dfrac{391}{87}$.

**Solution:** $x = \frac{67}{29},\ y = -\frac{391}{87}$.