Short multiplication formulas for squares

- Square of a sum
$$(a+b)^2 = a^2 + 2ab + b^2$$
- Square of a difference
$$(a-b)^2 = a^2 - 2ab + b^2$$
- Difference of squares
$$a^2 - b^2 = (a-b)(a+b)$$

Worth knowing

- $(a+b)^2 + (a-b)^2 = 2a^2 + 2b^2$
- $(a+b)^2 - (a-b)^2 = 4ab$

Warm-up activity

1. Fill the gap to get the true equation.

(a) $(x+\square)^2 = x^2 + 6x + 9$

(b) $(\square + 2t)^2 = 9 + \square + 4t^2$

(c) $(\square - 8)^2 = 8x^2 - \square + \square$

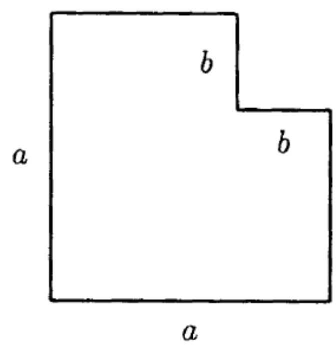
(d) $(\square - 3y)^2 = \square - 6y + \square$

(e) $(\square + \square)(\square - 3) = x^4 - \square$

(f) $(\square - \square)(\square + \square) = 16x^2 - 25$

Exercises

1. A piece of size $b \times b$ was cut from an $a \times a$ square. Cut the remaining part into pieces and combine the pieces into a rectangle with sides $a-b$ and $a+b$. (picture below)



2. Calculate the value of the following expression:

(a) $(x+1)(x-1) + (x-2)(x+2) - (x+3)(x-3)$ for $x = \sqrt{3}$,

(b) $(2x-1)^2 - (2x-1)(2x+1) - (2x+1)^2$ for $x = \sqrt{2}$.

(c) $(\sqrt{3}x - y)^2 - (x - \sqrt{3}y)^2$ for $x = \sqrt{6} + \sqrt{2}$, $y = \sqrt{6} - \sqrt{2}$

(d) $(2x-3y)(2x+3y) - (2x-3y)^2$ for $x = \sqrt{\sqrt{10} - 3}$, $y = \sqrt{\sqrt{10} + 3}$

3. Prove that for every natural number n, the number:

   (a) $(n+1)^2 - n^2$ is the odd number.

   (b) $(2n+1)^2$ is the odd number.

   (c) $\left(n+\frac{1}{2}\right)^2 - \left(n-\frac{1}{2}\right)^2$ is the even number.

   (d) $n^3 - n$ is divisible by 6.

4. Compute the total surface area of the cube whose the length of the edge is eqaul to a:

   (a) $a = 2 + \sqrt{5}$

   (b) $a = 2\sqrt{3} - 1$

   (c) $a = \sqrt{6} + \sqrt{2}$

5. Compute $(a+b+c)^2$. (Hint: Use a square with side length $a+b+c$)

6. Prove that:

   (a) if $c \neq 0$ and $(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab$, then $a = -b$.

   (b) if $b \neq 0$ and $(a-b+c)^2 = a^2 + c^2 - 2ab + 2ac$, then $a = 2c$.