# 1.7 Square root

1. `Definition`

   Given non-negative numbers $a, b$.

   $$\text{The square root of } a \text{ is the number } b \text{ such that } b^2 = a.$$

   Meaning that the square root of a number is the inverse operation of squaring a number. You can also think of a sqare root of a number as a factor of a number which when multiplied it by itself gives the original number.

   EXAMPLES:

   (a) $\sqrt{4} = 2$, because $2^2 = 4$ or $2 \cdot 2 = 4$.
   (b) $\sqrt{144} = 12$, because $12^2 = 144$ or $12 \cdot 12 = 144$.
   (c) $\sqrt{\frac{9}{64}} = \frac{3}{8}$, because $\left(\frac{3}{8}\right)^2 = \frac{9}{64}$.
   (d) $\sqrt{0.25} = 0.5$

   To find the square root of a natural number that is a perfect square, we can use one of the method given below.

   (a) **Repeated Subtraction Method**
   Subtract the consecutive odd numbers from the number for which we are finding the square root. We repeat the process until we get 0. The number of times we subtract is the square root of the given number. This method works only for perfect square numbers. Let us find the square root of 25 using this method.
   $25 - \mathbf{1} = 24$,
   $24 - \mathbf{3} = 21$,
   $21 - \mathbf{5} = 16$,
   $16 - \mathbf{7} = 9$,
   $9 - \mathbf{9} = 0$.
   We did the subtraction **five** times, which means that $\sqrt{25} = 5$.

   (b) **Prime Factorization Method**
   Prime factorization of any number means to represent that number as a product of prime numbers. To find the square root of a given number through the prime factorization method, we follow the steps given below:
   - **Step 1:** Divide the given number into its prime factors.
   - **Step 2:** Form pairs of factors such that both factors in each pair are equal.
   - **Step 3:** Take one factor from the pair.
   - **Step 4:** Find the product of the factors obtained by taking one factor from each pair.
   - **Step 5:** That product is the square root of the given number.

   Let us find the square root of 144 by this method. Lets take $\sqrt{324}$ and make prime factorization

   | | |
   |---|---|
   | 324 | 2 |
   | 162 | 2 |
   | 81 | 3 |
   | 27 | 3 |
   | 9 | 3 |
   | 3 | 3 |
   | 1 | |

   Thus $324 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = (2 \cdot 2) \cdot (3 \cdot 3) \cdot (3 \cdot 3) = 2^2 \cdot 3^2 \cdot 3^2 = (2 \cdot 3 \cdot 3)^2 = 18^2$, therefore $\sqrt{324} = 18$.

2. `Operations on square roots`

   - $\sqrt{x \cdot y} = x \cdot y$
   - $\sqrt{\frac{x}{y}} = \frac{x}{y}$

   How to simplify the root of a natural number that is not a perfect square? For example $\sqrt{180}$. First of all factorize the number 180.

   | | |
   |---|---|
   | 180 | 2 |
   | 90 | 2 |
   | 45 | 3 |
   | 15 | 3 |
   | 5 | 5 |
   | 1 | |

   Thus $180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = (2 \cdot 2) \cdot (3 \cdot 3) \cdot 5 = 2^2 \cdot 3^2 \cdot 5 = (2 \cdot 3)^2 \cdot 5 = 6^2 \cdot 5$.
   Now taking the square root of 180 we can notice that $\sqrt{180} = \sqrt{36 \cdot 5} = \sqrt{36} \cdot \sqrt{5} = 6\sqrt{5}$.

3. `Rationalizing the denominator`
   It is the process of eliminating irrational numbers from the denominator of a fraction, resulting in a denominator that contains only rational numbers. This is done by multiplying both the numerator and the denominator by a factor that will eliminate the irrational term in the denominator.

   EXAMPLE:

$$\frac{3}{\sqrt{2}} = \frac{3 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{3\sqrt{2}}{2}.$$

## EXERCISES

1. Write all natural numbers between given numbers:

   (a) $\sqrt{3}$ i $\sqrt{33}$ (b) $\sqrt{11}$ i $\sqrt{145}$ (c) $\sqrt{82}$ i $\sqrt{401}$

2. Simplify the radical.

   (a) $\sqrt{18}$ (c) $\sqrt{54}$ (e) $\sqrt{175}$ (g) $\sqrt{216}$
   (b) $\sqrt{50}$ (d) $\sqrt{96}$ (f) $\sqrt{252}$ (h) $\sqrt{450}$

3. Write the number in the form of $a\sqrt{b}$.

   (a) $\sqrt{72}+3\sqrt{2}$ (c) $\sqrt{18}+\sqrt{98}$ (e) $\sqrt{18}+\sqrt{72}+\sqrt{242}$
   (b) $\sqrt{32}-3\sqrt{18}$ (d) $\sqrt{48}-\sqrt{27}$ (f) $\sqrt{800}+\sqrt{242}-\sqrt{162}$

4. Rationalize denominators of the following radicals.

   (a) $\frac{5}{\sqrt{2}}$ (c) $\frac{\sqrt{5}}{\sqrt{3}}$ (e) $\frac{3}{5\sqrt{6}}$
   (b) $\frac{1}{\sqrt{3}}$ (d) $\frac{6}{5\sqrt{2}}$ (f) $\frac{5}{\sqrt{2}}$